QP CODE: 22001759

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Reg No : .....

Name : .....

M Sc DEGREE (CSS) EXAMINATION, AUGUST 2022

Fourth Semester

Elective - ME800402 - ALGORITHMIC GRAPH THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

E30B45EC

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define isomorphism of two graphs. Give an example of a graph G of order 5 such that $G \cong \bar{G}$.
2. Define join of two graphs. Find $P_3 + K_2$.
3. Differentiate between stack and queue
4. Define a tree. Draw all trees of order 4.
5. Define a complete $m - ary$ tree. Give an example
6. Write down the steps in Kruskal's Algorithm
7. Let f be a flow in a network N with underlyimg digraph D and capacity function c. Define an $f-$augmenting semi-path in D.
8. Determine $\lambda(K_{m,n})$ and $\kappa(K_{m,n})$ where $1 \leq m \leq n$.
9. *Define a 1-factor of a graph G. Give a 1-factorization of* $K_{3,3}$
10. *Find k and r in a BIBD with parameters* $b = 14, v = 7$ *and* $\lambda = 2$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. A connected graph G contains three distinct vertices u, v *and* w with the property that every $u - w$ path in G contains v. Show that v is a cut vertex.

12. Write sequential search algorithm and determine its complexity.

13. (a) Define the distance function on a graph G.
    (b) Write an algorithm to find the distance $d(u, v)$ for a fixed vertex u and every vertex v of a graph G.

14. Prove that the center of every tree is isomorphic to K_1 or K_2

15. Show that the value of the flow f in a network N equals the net flow into the sink t of N.

16. State and prove a necessary and sufficient condition that a graph G is $n-$connected for $n \geq 1$

17. *Prove that a maximum matching of the $n-$ cube contains 2^{n-1} edges*

18. *Let l be a feasible vertex labeling of a weighted complete bipartite graph G.(1) Define the sets E_l and H_l (2)If H_l contains a perfect matching M' then prove that M' is a maximum weight matching of G*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a)A sequence $s : d_1, d_2, \ldots\ldots d_p$ of non- negative integer,with $d_1 \geq d_2 \geq \ldots\ldots\ldots \geq d_p$, where $p \geq 2$ and $d_1 \geq 1$, is graphical if and only if the sequence

    $s_1 : d_2 - 1, d_3 - 1, \ldots . d_{d_1+1} - 1, d_{d_1+2}, d_{d_1+3} \ldots\ldots . d_p$ is graphical.

    b) Determine whether the following sequence is graphical and , if so, find a graph having s as its degree sequence.

    $s : 5, 5, 5, 4, 3, 2, 2, 0$

20. Explain Breadth First Search Algorithm using an example. Find its complexity.

21. Let N be a network with underlying digraph D. Prove that a flow f in $N$is a maximum flow if and only if there is no $f-$augmenting semi path in D.

22. *Explain the Kuhn Munkers algorithm to find a maximum weight perfect matching in a weighted complete bipartite graph G*

(2×5=10 weightage)